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Propagation of dark solitons in a system of coupled higher-order nonlinear Schrödinger equations

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Abstract

In this paper, we consider the higher-order linear and nonlinear effects in optics and analyse a coupled system of higher-order nonlinear Schrödinger equations to identify the conditions for dark-soliton propagation through Painlevé analysis, to supplement the known bright-soliton conditions. We also construct the explicit Lax pair and the Hirota bilinear form is used to generate one and two dark solitons.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Hasegawa and Tappert [1] predicted an ingenious method for obtaining a bit rate of gigabits s^{-1} in nonlinear optical fibres by employing the concept of solitons that was known to be appropriate for many other nonlinear systems. Optical solitons occur as a result of exact balance between the linear effect, i.e., the second-order dispersion which broadens the pulse, and the nonlinear effect, i.e., the Kerr effect which contracts the pulse. This kind of lossless solitary-wave propagation happens in the anomalous dispersion regime, while there are solitons of another kind, known as dark solitons [2], in the normal dispersion regime. They appear as an intensity dip in the constant background. These bright- and dark-soliton systems are both governed by the nonlinear Schrödinger equation (NLSE), which is one of the most well-known completely integrable systems in soliton theory, with only a sign change in the group velocity dispersion parameter. In this model, the interaction of the two polarizations involved in the soliton wave is ignored; this nonlinear interaction arises as a result of the tensor nature of the $\chi^{(3)}$-nonlinearity. However, in order to increase the bit rate and achieve wavelength-division multiplexing (WDM) that is utilized to transmit along more channels, or pulse propagation in birefringent fibres, one has to consider coupled systems. Solitary-wave analyses of such coupled systems in both isotropic and birefringent fibres are plentiful in the literature [3–7]. In 1974, Manakov proposed a coupled version of the NLSE (CNLSE) by considering the left- and right-polarized modes of the propagating electromagnetic wave in the following form [4]:

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$$\begin{aligned} \mathrm{i}q_{1t} \pm \tfrac{1}{2}q_{1xx} + (|q_1|^2 + |q_2|^2)q_1 &= 0 \\ \mathrm{i}q_{2t} \pm \tfrac{1}{2}q_{2xx} + (|q_1|^2 + |q_2|^2)q_2 &= 0 \end{aligned} \tag{1}$$

where q is the complex amplitude of the pulse envelope, x and t represent the spatial and temporal coordinates and the + or − signs before the dispersive terms denote the anomalous and normal dispersive regimes respectively. In the anomalous dispersive regime, this system possesses a bright-soliton solution and in the normal dispersive regime it possesses a dark-soliton solution. Recently, the N-soliton solutions for the above cases have been reported and also the inelastic collision of bright solitons has been analysed [8].

From extensive experimental work pioneered by Mollenauer *et al* [9], it was realized that in the picosecond range, the propagation of optical pulses is affected by higher-order effects such as third-order dispersion (TOD), self-steepening (SS) and stimulated Raman scattering (SRS) and hence these are included in the analysis of soliton propagation and the governing equation is the higher-order nonlinear Schrödinger equation (HNLSE) [10, 11]. Tasgal and Potasek [12] have analysed the coupled version of the Hirota equation which includes TOD and SS. In a similar manner, we have proposed an integrable version of the coupled higher-order nonlinear Schrödinger equation (CHNLSE) that admits bright-soliton propagation by including TOD, SS and SRS [13, 14]. Recently, a complete Painlevé analysis of the CHNLSE system has been carried out by Sakovich and Tsuchida [15], who identified some new integrable cases in addition to the already known cases. Here, it should be mentioned that Mihalache *et al* [16] have predicted that Painlevé analysis of the HNLSE could be extended to the dark-soliton case too.

It is well known that the effect of TOD is a splitting of higher-order solitons and that SS gives the pulse a very narrow width in the course of the propagation, because of which the peak of the pulse will travel slower than the wings. The inelastic Raman scattering is due to the delayed response of the medium which forces the pulse to undergo a frequency shift which is known as the self-frequency shift. However, analyses of dark solitons in coupled systems are scarce, even though in certain regards, such as inherent stability and reduction of jitter, dark solitons are preferred to bright solitons [17]. In a previous paper, we analysed dark-soliton propagation in the coupled Hirota system [18] which excludes SRS. We have generated one-dark-soliton and two-dark-soliton solutions by means of Hirota's bilinear form. For this system, Park and Shin [19] have constructed the Bäcklund transformation and analysed the dark–dark, bright–dark and bright–bright pairs of solutions. Here, it should be mentioned that Radhakrishnan and Lakshmanan [20] have analysed dark-soliton propagation in another coupled higher-order system which is known as the extended NLSE system. However, this system is not integrable from the point of view of Painlevé analysis [21]. By including the important effect of SRS, which is responsible for the self-frequency shift, in this paper, we have attempted to study the dark solitons in the CHNLSE system. We have not included the effect of group velocity mismatch between two coupled waves, which gives rise to the walking solitons studied extensively by Mihalache and co-workers [22].

First, we provide a common Lax pair for both the bright- and dark-soliton versions of the coupled NLSE system. As the bright- and dark-soliton cases for this system are well known, we proceed to the case of the CHNLSE. Through Painlevé analysis, we identify the integrable version of the CHNLSE for dark-soliton propagation. We explicitly construct the Lax pair for this particular system and the one-dark-soliton and two-dark-soliton solutions are generated by means of Hirota's bilinear form.

2. The Lax pair for dark solitons in the CNLSE system

The complete integrability of a nonlinear system is ensured by the Lax pair and one can obtain N-soliton solutions by means of the inverse scattering transform method once the Lax pair is known. In this paper, we follow the AKNS formalism [23] to obtain the Lax pair. The linear eigenvalue problem for optical solitons in the CNLSE system can be constructed as follows:

$$\begin{aligned} \Psi_x &= U\Psi \\ \Psi_t &= V\Psi \qquad \text{where} \quad \Psi = (\Psi_1\Psi_2\Psi_3)^{\mathrm{T}}. \end{aligned} \tag{2}$$

Here, the Lax operators U and V are given in the form

$$U = \begin{pmatrix} -\mathrm{i}\lambda/2 & -\mu q_1 & -\mu q_2 \\ \mu q_1^* & \mathrm{i}\lambda/2 & 0 \\ \mu q_2^* & 0 & \mathrm{i}\lambda/2 \end{pmatrix}$$

$$\begin{aligned} V = \lambda^2 \begin{pmatrix} -\mathrm{i}\bar{\mu}/2\mu & 0 & 0 \\ 0 & \mathrm{i}\bar{\mu}/2\mu & 0 \\ 0 & 0 & \mathrm{i}\bar{\mu}/2\mu \end{pmatrix} + \lambda \begin{pmatrix} 0 & -\bar{\mu}q_1 & -\bar{\mu}q_2 \\ -\bar{\mu}q_1^* & 0 & 0 \\ -\bar{\mu}q_2^* & 0 & 0 \end{pmatrix} \\ + \begin{pmatrix} \mathrm{i}\mu\bar{\mu}(|q_1|^2+|q_2|^2) & -\mathrm{i}\bar{\mu}q_{1x} & -\mathrm{i}\bar{\mu}q_{2x} \\ \mathrm{i}\bar{\mu}q_{1x}^* & \mathrm{i}\mu\bar{\mu}(|q_1|^2+|q_2|^2) & 0 \\ \mathrm{i}\bar{\mu}q_{2x}^* & 0 & \mathrm{i}\mu\bar{\mu}(|q_1|^2+|q_2|^2) \end{pmatrix} \end{aligned} \tag{3}$$

where λ is the eigenvalue parameter and μ and $\bar{\mu}$ are constants whose choices make the resultant equation either that for bright solitons or that for dark solitons as shown below.

Case 1. $\mu = \bar{\mu} = 1$. For this case, the compatibility condition $U_t - V_x + [U, V] = 0$ gives the NLSE for bright solitons, in the form

$$\begin{aligned} \mathrm{i}q_{1t} + q_{1xx} + 2(|q_1|^2 + |q_2|^2)q_1 &= 0 \\ \mathrm{i}q_{2t} + q_{2xx} + 2(|q_1|^2 + |q_2|^2)q_2 &= 0. \end{aligned} \tag{4}$$

Case 2. $\mu = \mathrm{i}$ and $\bar{\mu} = -\mathrm{i}$. For this case, the compatibility condition gives the NLSE for dark solitons:

$$\begin{aligned} \mathrm{i}q_{1t} - q_{1xx} + 2(|q_1|^2 + |q_2|^2)q_1 &= 0 \\ \mathrm{i}q_{2t} - q_{2xx} + 2(|q_1|^2 + |q_2|^2)q_2 &= 0. \end{aligned} \tag{5}$$

Recently, Park and Shin [24] have considered the Bäcklund transformation and generated the soliton solutions of equation (5).

3. Coupled higher-order nonlinear Schrödinger equations

The CHNLSE in the normal dispersion region is given in the following form:

$$\begin{aligned} q_{1t} &= -\mathrm{i}q_{1xx} + 2\mathrm{i}(|q_1|^2 + |q_2|^2)q_1 + \varepsilon\{q_{1xxx} + \alpha_1(|q_1|^2 + |q_2|^2)q_{1x} + \alpha_2 q_1(|q_1|^2 + |q_2|^2)_x\}, \\ q_{2t} &= -\mathrm{i}q_{2xx} + 2\mathrm{i}(|q_1|^2 + |q_2|^2)q_2 + \varepsilon\{q_{2xxx} + \alpha_1(|q_1|^2 + |q_2|^2)q_{2x} + \alpha_2 q_2(|q_1|^2 + |q_2|^2)_x\}, \end{aligned} \tag{6}$$

where q is the slowly varying amplitude of the pulse envelope and α_1 and α_2 are arbitrary constants. In the past few years, equation (6) has been investigated in detail for bright solitons. To the best of our knowledge, the dark-soliton solution for equation (6) has not been studied. A common feature of all soliton-possessing systems is their integrability. We follow the WTC procedure [25] to carry out the Painlevé analysis to identify the new integrable systems. This procedure emphasizes that a given partial differential equation (PDE) is integrable if its solutions are single valued for the movable-singularity manifold. Throughout this analysis, for simplicity, we have used Kruskal's reduced-manifold ansatz.

In order to carry out the Painlevé analysis, let us assume $q_1 = a$, $q_1^* = b$, $q_2 = c$ and $q_2^* = d$; thus equation (6) becomes

$$\begin{aligned}
a_t &= -\mathrm{i}a_{xx} + 2\mathrm{i}(ab+cd)a + \varepsilon\{a_{xxx} + \alpha_1(ab+cd)a_x + \alpha_2 a(ab+cd)_x\},\\
b_t &= \mathrm{i}a_{xx} - 2\mathrm{i}(ab+cd)b + \varepsilon\{b_{xxx} + \alpha_1(ab+cd)b_x + \alpha_2 b(ab+cd)_x\},\\
c_t &= -\mathrm{i}c_{xx} + 2\mathrm{i}(ab+cd)c + \varepsilon\{c_{xxx} + \alpha_1(ab+cd)c_x + \alpha_2 c(ab+cd)_x\},\\
d_t &= \mathrm{i}d_{xx} - 2\mathrm{i}(ab+cd)d + \varepsilon\{d_{xxx} + \alpha_1(ab+cd)d_x + \alpha_2 d(ab+cd)_x\}.
\end{aligned} \tag{7}$$

To determine the leading-order behaviour, we substitute in equation (7) $a \approx a_0\phi^p, b \approx b_0\phi^q, c \approx c_0\phi^r, d \approx d_0\phi^s$ where p, q, r, s are negative integers and we obtain

$$p = q = r = s = -1 \qquad \text{and} \qquad a_0b_0 + c_0d_0 = -6/(3\alpha_1 + 2\alpha_2). \tag{8}$$

To find the resonances, we make the substitutions

$$\begin{aligned}
a &= a_0\phi^{-1} + a_j\phi^{j-1},\\
b &= b_0\phi^{-1} + b_j\phi^{j-1},\\
c &= c_0\phi^{-1} + c_j\phi^{j-1},\\
d &= d_0\phi^{-1} + d_j\phi^{j-1}.
\end{aligned} \tag{9}$$

Collecting the coefficients of ϕ^{j-4} and solving for the resultant determinant, the resonances are obtained as

$$j = -1, 0, 0, 0, 2, 2, 3, 4, 4, 4, 3 \pm 2\sqrt{\frac{\alpha_1 - \alpha_2}{\alpha_1 + 2\alpha_2}}. \tag{10}$$

The resonance at $j = -1$ corresponds to the arbitrariness of the singular manifold and the arbitrariness at $j = 0, 0, 0$ is verified from equation (8) which shows that of the four coefficients a_0, b_0, c_0 and d_0, any three coefficients are arbitrary. From the resonance analysis, it can be clearly seen that the resonances will be integers when $\alpha_1 = 2\alpha_2$. The arbitrariness at other resonances can be checked by substituting the full Laurent series into equation (7).

From the coefficient of (ϕ^{-3}, ϕ^{-3}, ϕ^{-3}, ϕ^{-3}), it can be shown that

$$\begin{aligned}
a_1 &= \frac{2\mathrm{i}(a_0b_0 + c_0d_0 - 1)}{\alpha_1\varepsilon b_0},\\
b_1 &= \frac{-2\mathrm{i}(a_0b_0 + c_0d_0 - 1)}{\alpha_1\varepsilon a_0},\\
c_1 &= \frac{2\mathrm{i}(a_0b_0 + c_0d_0 - 1)}{\alpha_1\varepsilon d_0},\\
d_1 &= \frac{-2\mathrm{i}(a_0b_0 + c_0d_0 - 1)}{\alpha_1\varepsilon c_0}.
\end{aligned} \tag{11}$$

Similarly, from the coefficient of (ϕ^{-2}, ϕ^{-2}, ϕ^{-2}, ϕ^{-2}), we can show that three of the four coefficients a_2, b_2, c_2 and d_2 are arbitrary, which corresponds to the resonance at $j = 2, 2, 2$. From the higher powers of ϕ, one can show that in order to prove the existence of a sufficient number of arbitrary functions, the values of the parameters α_1 and α_2 should be equal to -6 and -3 respectively. It is interesting to note that the Painlevé analysis gives the same resonance values for the bright-soliton case also. Thus, with just the sign changes, the integrability for dark-soliton case is provided. Also, when $\alpha_2 = 0$, the system reduces to the Hirota case, for which there exist dark solitons of similar type [18]. Hence, the Painlevé analysis suggests that dark-soliton solutions are possible for the CHNLSE system. As the next logical step, we proceed to establish the complete integrability properties of this system, such as the Lax pair, Hirota's bilinear form and soliton solutions.

4. The Lax pair for the CHNLSE system

The linear eigenvalue problem for the CHNLSE can be presented in the following form:

$$\begin{aligned}\Psi_x &= U\Psi\\ \Psi_t &= V\Psi \qquad \text{where} \quad \Psi = (\Psi_1\Psi_2\Psi_3\Psi_4\Psi_5)^{\mathrm{T}}.\end{aligned} \tag{12}$$

The U- and V-matrices are found to be in the following form:

$$U = \begin{pmatrix} -\mathrm{i}\lambda/2 & -k_1q_1 & -k_1r_1 & -k_1q_2 & -k_1r_2\\ k_1q_1^* & \mathrm{i}\lambda/2 & 0 & 0 & 0\\ k_1r_1^* & 0 & \mathrm{i}\lambda/2 & 0 & 0\\ k_1q_2^* & 0 & 0 & \mathrm{i}\lambda/2 & 0\\ k_1r_2^* & 0 & 0 & 0 & \mathrm{i}\lambda/2 \end{pmatrix} \tag{13}$$

and

$$\begin{aligned} V = {} & \lambda^3 \begin{pmatrix} \mathrm{i}\varepsilon/2 & 0 & 0 & 0 & 0\\ 0 & -\mathrm{i}\varepsilon/2 & 0 & 0 & 0\\ 0 & 0 & -\mathrm{i}\varepsilon/2 & 0 & 0\\ 0 & 0 & 0 & -\mathrm{i}\varepsilon/2 & 0\\ 0 & 0 & 0 & 0 & -\mathrm{i}\varepsilon/2 \end{pmatrix}\\ & + \lambda^2 \begin{pmatrix} A_2 & \varepsilon k_1q_1 & \varepsilon k_1r_1 & \varepsilon k_1q_2 & \varepsilon k_1r_2\\ -\varepsilon k_1q_1^* & -A_2 & 0 & 0 & 0\\ -\varepsilon k_1r_1^* & 0 & -A_2 & 0 & 0\\ -\varepsilon k_1q_2^* & 0 & 0 & -A_2 & 0\\ -\varepsilon k_1r_2^* & 0 & 0 & 0 & -A_2 \end{pmatrix}\\ & + \lambda \begin{pmatrix} -\mathrm{i}\varepsilon k_1^2\sum_{j=1}^{2}(|q_j|^2+|r_j|^2) & \begin{smallmatrix}\mathrm{i}\varepsilon k_1q_{1x}\\ -2\mathrm{i}A_2k_1q_1\end{smallmatrix} & \begin{smallmatrix}\mathrm{i}\varepsilon k_1r_{1x}\\ -2\mathrm{i}A_2k_1r_1\end{smallmatrix} & \begin{smallmatrix}\mathrm{i}\varepsilon k_1q_{2x}\\ -2\mathrm{i}A_2k_1q_2\end{smallmatrix} & \begin{smallmatrix}\mathrm{i}\varepsilon k_1r_{2x}\\ -2\mathrm{i}A_2k_1r_2\end{smallmatrix}\\ \mathrm{i}\varepsilon k_1q_{1x}^* + 2\mathrm{i}A_2k_1q_1^* & \mathrm{i}\varepsilon k_1^2|q_1|^2 & \mathrm{i}\varepsilon k_1^2q_1^*r_1 & \mathrm{i}\varepsilon k_1^2q_1^*q_2 & \mathrm{i}\varepsilon k_1^2q_1^*r_2\\ \mathrm{i}\varepsilon k_1r_{1x}^* + 2\mathrm{i}A_2k_1r_1^* & \mathrm{i}\varepsilon k_1^2r_1^*q_1 & \mathrm{i}\varepsilon k_1^2|r_1|^2 & \mathrm{i}\varepsilon k_1^2r_1^*q_2 & \mathrm{i}\varepsilon k_1^2r_1^*r_2\\ \mathrm{i}\varepsilon k_1q_{2x}^* + 2\mathrm{i}A_2k_1q_2^* & \mathrm{i}\varepsilon k_1^2q_2^*q_1 & \mathrm{i}\varepsilon k_1^2q_2^*r_1 & \mathrm{i}\varepsilon k_1^2|q_2|^2 & \mathrm{i}\varepsilon k_1^2q_2^*r_2\\ \mathrm{i}\varepsilon k_1r_{2x}^* + 2\mathrm{i}A_2k_1r_2^* & \mathrm{i}\varepsilon k_1^2r_2^*q_1 & \mathrm{i}\varepsilon k_1^2r_2^*r_1 & \mathrm{i}\varepsilon k_1^2r_2^*q_2 & \mathrm{i}\varepsilon k_1^2|r_2|^2 \end{pmatrix}\\ & + \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15}\\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25}\\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35}\\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45}\\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{pmatrix} \end{aligned}$$

where

$$M_{11} = -2A_2k_1^2\sum_{j=1}^{2}(|q_j|^2+|r_j|^2) - \varepsilon k_1^2\sum_{j=1}^{2}(q_jq_{jx}^* - q_{jx}q_j^* + r_jr_{jx}^* - r_{jx}r_j^*),$$

$$M_{12} = -\varepsilon k_1q_{1xx} + 2A_2k_1q_{1x} - 2\varepsilon k_1^3q_1\sum_{j=1}^{2}(|q_j|^2+|r_j|^2),$$

$$M_{13} = -\varepsilon k_1r_{1xx} + 2A_2k_1r_{1x} - 2\varepsilon k_1^3r_1\sum_{j=1}^{2}(|q_j|^2+|r_j|^2),$$

$$
\begin{aligned}
M_{14} &= -\varepsilon k_1 q_{2xx} + 2A_2 k_1 q_{2x} - 2\varepsilon k_1^3 q_2 \sum_{j=1}^{2} (|q_j|^2 + |r_j|^2), \\
M_{15} &= -\varepsilon k_1 r_{2xx} + 2A_2 k_1 r_{2x} - 2\varepsilon k_1^3 r_2 \sum_{j=1}^{2} (|q_j|^2 + |r_j|^2), \\
M_{21} &= \varepsilon k_1 q_{1xx}^* + 2A_2 q_{1x}^* + 2\varepsilon k_1^3 q_1^* \sum_{j=1}^{2} (|q_j|^2 + |r_j|^2), \\
M_{22} &= -\varepsilon k_1^2 (q_1^* q_{1x} - q_{1x}^* q_1) + 2A_2 k_1^2 q_1^* q_1, \\
M_{23} &= -\varepsilon k_1^2 (q_1^* r_{1x} - q_{1x}^* r_1) + 2A_2 k_1^2 q_1^* r_1, \\
M_{24} &= -\varepsilon k_1^2 (q_1^* q_{2x} - q_{1x}^* q_2) + 2A_2 k_1^2 q_1^* q_2, \\
M_{25} &= -\varepsilon k_1^2 (q_1^* r_{2x} - q_{1x}^* r_2) + 2A_2 k_1^2 q_1^* r_2, \\
M_{31} &= \varepsilon k_1 r_{1xx}^* + 2A_2 r_{1x}^* + 2\varepsilon k_1^3 r_1^* \sum_{j=1}^{2} (|q_j|^2 + |r_j|^2), \\
M_{32} &= -\varepsilon k_1^2 (r_1^* q_{1x} - r_{1x}^* q_1) + 2A_2 k_1^2 r_1^* q_1, \\
M_{33} &= -\varepsilon k_1^2 (r_1^* r_{1x} - r_{1x}^* r_1) + 2A_2 k_1^2 r_1^* r_1, \\
M_{34} &= -\varepsilon k_1^2 (r_1^* q_{2x} - r_{1x}^* q_2) + 2A_2 k_1^2 r_1^* q_2, \\
M_{35} &= -\varepsilon k_1^2 (r_1^* r_{2x} - r_{1x}^* r_2) + 2A_2 k_1^2 r_1^* r_2, \\
M_{41} &= \varepsilon k_1 q_{2xx}^* + 2A_2 k_1 q_{2x}^* + 2\varepsilon k_1^3 q_2^* \sum_{j=1}^{2} (|q_j|^2 + |r_j|^2), \\
M_{42} &= -\varepsilon k_1^2 (q_2^* q_{1x} - q_{2x}^* q_1) + 2A_2 k_1 q_2^* q_1, \\
M_{43} &= -\varepsilon k_1^2 (q_2^* r_{1x} - q_{2x}^* r_1) + 2A_2 k_1^2 q_2^* r_1, \\
M_{44} &= -\varepsilon k_1^2 (q_2^* q_{2x} - q_{2x}^* q_2) + 2A_2 k_1^2 q_2^* q_2, \\
M_{45} &= -\varepsilon k_1^2 (q_2^* r_{2x} - q_{2x}^* r_2) + 2A_2 k_1^2 q_2^* r_2, \\
M_{51} &= \varepsilon k_1 r_{2xx}^* + 2A_2 k_1 r_{2x}^* + 2\varepsilon k_1^3 r_2^* \sum_{j=1}^{2} (|q_j|^2 + |r_j|^2), \\
M_{52} &= -\varepsilon k_1^2 (r_2^* q_{1x} - r_{2x}^* q_1) + 2A_2 k_1 r_2^* q_1, \\
M_{53} &= -\varepsilon k_1^2 (r_2^* r_{1x} - r_{2x}^* r_1) + 2A_2 k_1^2 r_2^* r_1, \\
M_{54} &= -\varepsilon k_1^2 (r_2^* q_{2x} - r_{2x}^* q_2) + 2A_2 k_1^2 r_2^* q_2, \\
M_{55} &= -\varepsilon k_1^2 (r_2^* r_{2x} - r_{2x}^* r_2) + 2A_2 k_1^2 r_2^* r_2,
\end{aligned}
$$

with

$$
r_1 = \mathrm{e}^{\mathrm{i}\Theta} q_1^*, \qquad r_2 = \mathrm{e}^{\mathrm{i}\Theta} q_2^* \qquad \text{and} \qquad \Theta(x,t) = \tfrac{2}{3}\left(x + \tfrac{2}{9}t\right).
$$

Here, k_1 and A_2 are constants; suitable choice of their values gives either the bright- or dark-soliton version of the CHNLSE system. The compatibility condition $U_t - V_x + [U, V] = 0$ gives rise to the following CHNLSEs:

$$
\begin{aligned}
& k_1 q_{1t} + 2A_2 k_1 q_{1xx} + 4k_1^3 A_2 (|q_1|^2 + |q_2|^2) q_1 \\
&\qquad - \mathrm{i}\varepsilon[-\mathrm{i}k_1 q_{1xxx} - 6\mathrm{i}k_1^3 (|q_1|^2 + |q_2|^2) q_{1x} - 3\mathrm{i}k_1^3 (|q_1|^2 + |q_2|^2)_x q_1] = 0, \\
& k_1 q_{2t} + 2A_2 k_1 q_{2xx} + 4k_1^3 A_2 (|q_1|^2 + |q_2|^2) q_2 \\
&\qquad - \mathrm{i}\varepsilon[-\mathrm{i}k_1 q_{2xxx} - 6\mathrm{i}k_1^3 (|q_1|^2 + |q_2|^2) q_{2x} - 3\mathrm{i}k_1^3 (|q_1|^2 + |q_2|^2)_x q_2] = 0.
\end{aligned}
\tag{14}
$$

It has been found that the above equations give the bright-soliton CHNLSE for the choice

$$k_1 = 1, \qquad A_2 = -\mathrm{i}/2,$$

while for the choice

$$k_1 = \mathrm{i}, \qquad A_2 = \mathrm{i}/2,$$

one can obtain the dark-soliton equation.

Thus, by constructing the Lax pair for this system, we have proved the complete integrability of the same. From this Lax pair, one can obtain the Bäcklund transformation and generate multisoliton solutions. However, for the dark-soliton case, this method is a very tedious process. Hence, we proceed further to obtain the one- and two-soliton solutions by using Hirota's bilinear technique, which is comparatively simple and straightforward, as established in the next section.

5. Dark-soliton solutions

Hirota's bilinear technique [26] is a novel method for generating soliton solutions and constructing N-soliton solutions for nonlinear partial differential equations, even though it involves *ad hoc* assumptions for obtaining the necessary transformations to get the bilinear form of a particular equation. In many cases, such transformations can be sought by applying the Painlevé singularity structure analysis. In our case, it would be rather convenient to transform the CHNLSEs into a set of complex modified KdV equations with the help of the following transformations:

$$\begin{aligned} q_1(x,t) &= Q_1(Z,T)\exp\left[\mathrm{i}\left(\frac{Z}{3\varepsilon} - \frac{T}{27\varepsilon^2}\right)\right], \\ q_2(x,t) &= Q_2(Z,T)\exp\left[\mathrm{i}\left(\frac{Z}{3\varepsilon} - \frac{T}{27\varepsilon^2}\right)\right], \\ T &= t, \qquad Z = x + \frac{t}{3\varepsilon}. \end{aligned} \tag{15}$$

Then, equation (6) becomes

$$\begin{aligned} &Q_{1T} - \varepsilon[Q_{1ZZZ} - 6(|Q_1|^2 + |Q_2|^2)Q_{1Z} - 3Q_1(|Q_1|^2 + |Q_2|^2)_Z] = 0, \\ &Q_{2T} - \varepsilon[Q_{2ZZZ} - 6(|Q_1|^2 + |Q_2|^2)Q_{2Z} - 3Q_2(|Q_1|^2 + |Q_2|^2)_Z] = 0. \end{aligned} \tag{16}$$

The Hirota bilinear form for the CHNLSE can be constructed by applying the transformation for the field variables as follows:

$$Q_1(Z,T) = \frac{G(Z,T)}{F(Z,T)}, \qquad Q_2(Z,T) = \frac{H(Z,T)}{F(Z,T)}, \tag{17}$$

where $G(Z,T)$ and $H(Z,T)$ are complex functions and $F(Z,T)$ is a real function. Using equation (17), the decoupled bilinear forms of equation (16) are given as

$$\begin{aligned} &(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)G\cdot F = 0, \\ &(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)H\cdot F = 0, \\ &(D_Z^2 - \lambda)F\cdot F = -4(|G|^2 + |H|^2), \\ &D_Z G^*\cdot G = D_Z H^*\cdot H = 0, \end{aligned} \tag{18}$$

where λ is a constant to be determined and the Hirota bilinear operators D_x and D_t are defined as

$$D_x^m D_t^n G(x,t)\cdot F(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n G(x,t)F(x',t')\bigg|_{x=x',t=t'}. \tag{19}$$

To get one-soliton solutions, we assume

$$G = g_0(1 + \chi g_1), \qquad H = h_0(1 + \chi h_1), \qquad F = 1 + \chi f_1 \tag{20}$$

where g_0 and h_0 are complex constants and g_1, h_1 and f_1 are real functions. Substituting equation (20) in (19) and collecting the coefficients of χ^0, we get

$$\lambda = 4(|g_0|^2 + |h_0|^2). \tag{21}$$

The coefficient of χ leads to the following equations:

$$\begin{aligned}
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(1 \cdot f_1 + g_1 \cdot 1) = 0,\\
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(1 \cdot f_1 + h_1 \cdot 1) = 0,\\
&(D_Z^2 - \lambda)(1 \cdot f_1 + f_1 \cdot 1) + 8(|g_0|^2 + |h_0|^2) = 0.
\end{aligned} \tag{22}$$

The coefficient of χ^2 leads to the following equations:

$$\begin{aligned}
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(g_1 \cdot f_1) = 0,\\
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(h_1 \cdot f_1) = 0,\\
&(D_Z^2 - \lambda)(f_1 \cdot f_1) + 4[|g_0|^2 g_1^2 + |h_0|^2 h_1^2] = 0.
\end{aligned} \tag{23}$$

Equations (22), (23) suggest that they can be solved if we assume that

$$g_1 = h_1 - f_1 = -\exp[\omega_1 T + c_1 Z + \xi_1^{(0)}] \tag{24}$$

where

$$\omega_1 = \varepsilon c_1(c_1^2 - 3\lambda) \qquad \text{and} \qquad c_1^2 = 2\lambda = 8[|g_0|^2 + |h_0|^2].$$

Using equations (24) and (17), the one-dark-soliton solution of the cmKdV equation is obtained as

$$\begin{aligned}
Q_1 &= g_0 \tanh\left[\frac{1}{2}\left\{c_1\left(Z - \frac{c_1^2\varepsilon T}{2}\right) + \xi_1^{(0)}\right\}\right],\\
Q_2 &= h_0 \tanh\left[\frac{1}{2}\left\{c_1\left(Z - \frac{c_1^2\varepsilon T}{2}\right) + \xi_1^{(0)}\right\}\right].
\end{aligned} \tag{25}$$

Using the transformations (22), we can easily obtain the corresponding one-dark-soliton solution of the CHNLSE (6). This dark-soliton solution is plotted as shown in figure 1. It can be seen that the optical pulse retains its dark-soliton shape even in the presence of higher-order effects, characteristic of all soliton pulses. From here, we proceed to the next step of obtaining two-dark-soliton solutions, for which we assume

$$G = g_0(1 + \chi g_1 + \chi^2 g_2), \qquad H = h_0(1 + \chi h_1 + \chi^2 h_2), \qquad F = 1 + \chi f_1 + \chi^2 f_2 \tag{26}$$

where g_0, h_0 are complex constants and g_1, g_2, g_3, h_1, h_2, h_3, f_1 and f_2 are real functions.

The coefficient of χ^0 leads to equation (21). From the coefficient of χ, we get

$$\begin{aligned}
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(1 \cdot f_1 + g_1 \cdot 1) = 0,\\
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(1 \cdot f_1 + h_1 \cdot 1) = 0,\\
&(D_Z^2 - \lambda)(1 \cdot f_1 + f_1 \cdot 1) + 8[|g_0|^2 g_1 + |h_0|^2 h_1] = 0.
\end{aligned} \tag{27}$$

To solve these equations, we assume

$$g_1 = h_1 = P_1 \exp[\xi_1] + P_2 \exp[\xi_2] \qquad \text{and} \qquad f_1 = \exp[\xi_1] + \exp[\xi_2], \tag{28}$$

where $\xi_1 = \omega_1 T + c_1 Z + \xi_1^{(0)}$ and $\xi_2 = \omega_2 T + c_2 Z + \xi_2^{(0)}$ with

$$\omega_1 = \varepsilon c_1^3 - 3\varepsilon\lambda c_1 \qquad \text{and} \qquad \omega_2 = \varepsilon c_2^3 - 3\varepsilon\lambda c_2. \tag{29}$$

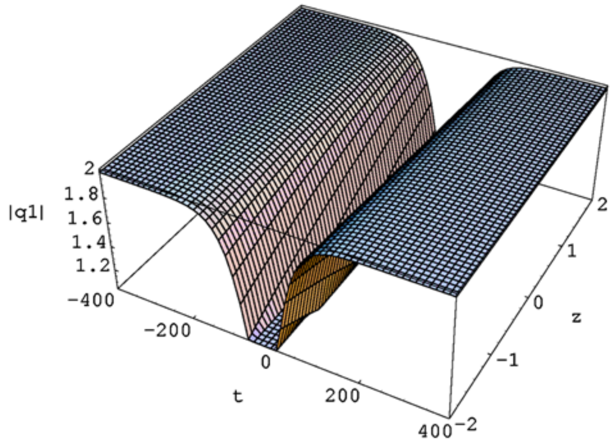

Figure 1. The one-soliton solution.

The values of P_1 and P_2 are found to be

$$P_1 = \frac{4[|g_0|^2 + |h_0|^2] - c_1^2}{4[|g_0|^2 + |h_0|^2]} \qquad \text{and} \qquad P_2 = \frac{4[|g_0|^2 + |h_0|^2] - c_2^2}{4[|g_0|^2 + |h_0|^2]}. \tag{30}$$

The coefficient of χ^2 leads to the following equations:

$$\begin{aligned}
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(1 \cdot f_2 + g_1 \cdot f_1 + g_2 \cdot 1) = 0,\\
&(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)(1 \cdot f_2 + h_1 \cdot f_1 + h_2 \cdot 1) = 0,\\
&(D_Z^2 - \lambda)(1 \cdot f_2 + f_1 \cdot f_1 + f_2 \cdot 1) + 2[2|g_0|^2 g_2 + |g_0|^2 g_1^2 + 2|h_0|^2 h_2 + |h_0|^2 h_1^2] = 0.
\end{aligned} \tag{31}$$

It can be shown that the above system of equations can be satisfied if we assume

$$g_2 = h_2 = A_{12} P_1 P_2 \exp[\xi_1 + \xi_2] \qquad \text{and} \qquad f_2 = A_{12} \exp[\xi_1 + \xi_2]. \tag{32}$$

The value of A_{12} is found to be

$$A_{12} = \frac{(P_2 - P_1)\{-(\omega_2 - \omega_1) + \varepsilon(c_2 - c_1)^3 - 3\varepsilon\lambda(c_2 - c_1)\}}{(1 - P_1 P_2)\{-(\omega_2 + \omega_1) + \varepsilon(c_2 + c_1)^3 - 3\varepsilon\lambda(c_2 + c_1)\}}. \tag{33}$$

The two-dark-soliton solution for the CHNLSE can be obtained by using the expressions for g_1, g_2, h_1, h_2, f_1 and f_2. This two-dark-soliton solution is plotted as shown in figure 2, which clearly shows that after collision, the pulses retain their shape with a slight change in their phase. Thus, in this paper, we have reported the dark-soliton version of the CHNLSEs and also obtained the one-dark-soliton and two-dark-soliton solutions for both equations using the Hirota bilinear method.

6. Conclusions

In this paper, we have discussed the dark solitons of CNLSEs and CHNLSEs. Using the AKNS formalism, we have given the Lax pair for the dark-soliton CNLSE system. Using the Painlevé analysis, for the dark-soliton CHNLSE system, we have obtained a new choice of parameters for the integrable case. It was found that the system admits dark-soliton propagation when the coefficients of SS and SRS are negative, −6 and −3, to be precise. The integrability of the above equation was also proved using the specific Lax pair. Indeed, this existence of dark solitons in the coupled higher-order nonlinear Schrödinger equations is confirmed, as we are able to obtain the one- and two-soliton solutions by means of Hirota's bilinear technique.

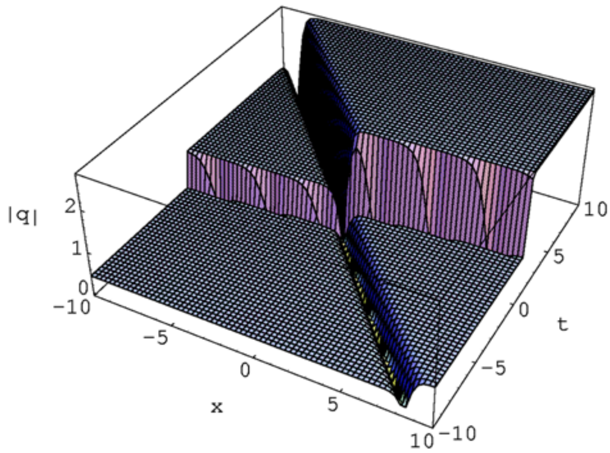

Figure 2. The two-soliton solution.

From the plots, it can be clearly seen that dark solitons exist for the CHNLSE system, as the subpicosecond optical pulses retain their dark-solitary-wave nature even in the presence of higher-order effects like TOD, SS and SRS. In this regard, they are very similar to the bright solitons. Moreover, the higher-order terms affect the velocity of these solitons, but otherwise leave their shape intact. From the plot for the two-soliton solution, we conclude that the presence of higher-order terms certainly influences the phase and velocity of dark solitons. Yet they maintain their inelastic behaviour since, after collision, they retain their shape and intensity with only a slight change in their phase. Hence, we conclude that dark solitons of this kind can be used to achieve all-optical communication links and in many other applications (switching etc), similarly to the dark solitons in other optical systems.

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